Chain Bankruptcy Size in Inter-bank Networks: the Effects of Asset Price Volatility and the Network Structure

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Abstract. One bankrupt of a certain bank can make another bank go bankrupt. This phenomenon is called chain bankruptcy. Chain bankruptcy is a kind of "systemic risk," a topic that has received a great deal of attention from researchers, recently. Here, we analyzed the effect of the asset price fluctuation and the inter-bank lending and borrowing network on chain bankruptcy by using an agent-based simulation. We found that: (1) As the rate of change in asset price grows, the total number of bankruptcies increases. On the other hand, when the rate of change in asset prices exceeds a certain value, the total number of bankruptcies became unvarying; (2) as the density of links increases, the total number of bankruptcies decreases, except when a certain situation occurs in core-periphery networks. These results suggest that factors causing bankruptcy are asset price fluctuations and the network structure of the inter-bank network.

Keywords: Systemic risk · Chain bankruptcy · Inter-bank networks.

1 Introduction

1.1 Introduction

The 2008 global financial crisis derived from a much smaller incident that occurred in an American company's housing loan department [2]. Moreover, the European sovereign debt crisis, which had a significant negative impact on Spain and Italy, began with a sharp downgrade of Greece's debt status by credit-rating companies[8].

The risk posed to the whole financial system caused by the bankruptcy of a single financial institution is called "systemic risk," and in recent years, the studies to promote systemic stability have attracted a great deal of attention.

1.2 Previous studies

Based on the above background, we examined the factors affecting a kind of systemic risk called chain bankruptcy in our study evaluating financial systemic stability.

Imakubo[3] and Hashimoto and Kurahashi[1][6] have researched systemic risk. Imakubo[3] focused on short-term financing and did not deal with chain bankruptcy. Hashimoto and Kurahashi[6] explained the causal dependence between bankruptcy and financial transaction networks and devised a means of forecasting the progress of chain bankruptcies. Those work, however, treated the price of risk assets such as stocks and bonds on the balance sheet as exogenous and did not consider the interaction between price fluctuation and systemic risk.

This study used agent simulations to examine the effect on chain bankruptcy of varying

1. the volatility of marketable assets that banks hold and
2. the type of inter-bank lending and borrowing network.

2 Model

2.1 Brief description of model

In our model, a certain bank is made to go bankrupt forcibly at the first moment of each trial. Under this condition, we count how many banks go bankrupt at the last moment. In this way, we examine situations in which the number of bankruptcies increases. We denote the bank made to go bankrupt at the beginning as the "start bank."

The procedure of each trial is as follows:

1. Initialize the settings
2. Buy or sell risk assets
3. Update the price of risk assets
4. Update the balance sheet of each bank
5. Judge whether banks become bankrupt or not, and deal with bankrupt banks
6. Repeat steps 2 to 5

In each trial of the simulation, Steps 2 to 5 are repeated 100 times. The model includes 100 banks. As city (large) banks account for 10% to 20% of the whole, the number of large banks is 10, while the number of smaller banks is 90. The inter-bank network is described in section 2.2. A large bank has more links in the inter-bank network than a smaller bank does. Moreover, the amount of assets that each bank holds rises as the number of links increases, as well[4].

2.2 Inter-bank network

Each bank borrows money from another bank and lends money to another bank. A network in which each bank corresponds to a node and an account is a link is called an inter-bank network. In the simulation, the inter-bank network is a directed graph to indicate whether a bank borrows or lends money. There is a 50% chance of a bank borrowing and a 50% chance of a bank lending. The three networks used in this model are described below.

The structure of inter-bank networks differs from country to country and from period to period, but most inter-bank networks have the same characteristics as the following three kinds of network.

1. core-periphery
2. scale-free
3. random

The outline of these networks and how to make them are described below.

Core-periphery networks

Characteristics of core-periphery networks According to Imakubo[3], the inter-bank networks of the short-term inter-bank market have the following characteristics.

1. a two-layer structure consisting of a core and periphery
2. all nodes in the core connect to every other node in the core
3. core is a hub to the periphery
4. the periphery consists of clusters

Graph 1 is a conceptual diagram of a core-periphery network.

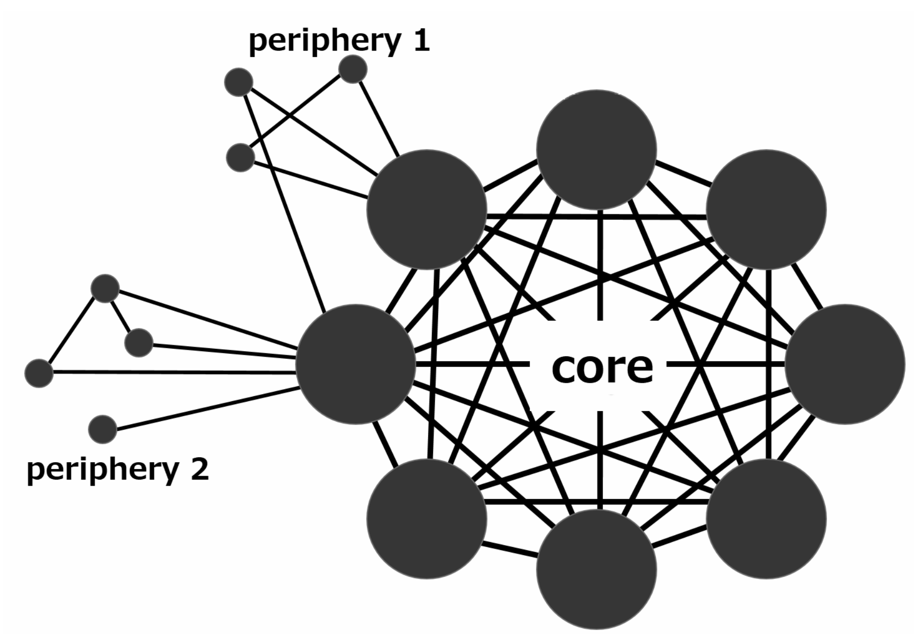

Fig. 1. Conceptual diagram of core-periphery network

How to make a core-periphery network To reproduce the features of core-periphery networks in our simulation, we perform the following steps:

1. Connect the nodes of the ten large banks to each other.
2. Divide the 90 smaller banks into ten groups.
3. Each large bank and each group of smaller banks made in step 2 are made to correspond to each other in a one-to-one fashion, and the large bank connects to each smaller bank in the group.
4. In each group of smaller banks, four smaller banks connect to each other and form a cluster.
5. Each bank connects to other banks which it does not unite in random order.

Step 1 corresponds to feature 2 above, while steps 2 and 3 correspond to features 1 and 3, and step 4 to feature 4.

Scale-free networks

Characteristics of scale-free networks Scale-free networks have two important characteristics[7].

- Growth: networks get larger over time as nodes are added to them.
- Preferential selection: Nodes that connect more to other nodes have a higher probability of connecting to new nodes.

According to above features, the number of links that each node has is variable. A small minority of nodes in the network, the hub, have a lot of links. On the other hand, most nodes have only a few links.

These characteristics are called scale-free and are observed in real world networks such as the Web [7].

How to make scale-free networks To create of scale-free networks and reproduce their features, we perform the following steps:

1. Connect all nodes of the ten large banks to each other.
2. Add nodes as smaller banks to the network made in step 1.

The procedure for adding nodes is as follows. Added nodes have a fixed number of links. These nodes connect to the existing network, and the probability of connecting to certain bank i,p_i, is determined by Eq. 1.

$$p_i = \frac{Link_i}{2 * LinkSum} \tag{1}$$

In this equation, $Link_i$ means the total number of links connected to bank i, and $LinkSum$ means the total number of links in the networks.

$LinkSum$ is doubled in the denominator of Eq. 1, because a link that connects bank A and bank B is counted twice (one link is considered to be a link of A connected to B and a link of B connected to A).

Equation 1 means that the more links a bank has, the higher the probability of it connecting to an added bank is. As a result, a lot of links are concentrated on a small number of nodes, and the scale-free property is realized. This addition is repeated 90 times. Each node is connected by roulette selection.

Random networks Random networks are those in which links are connected in random order. In this study, a Monte Carlo method is used to make random networks.

2.3 Risk assets

Brief description of market Each bank holds risk assets such as stocks and their price at a time point t, p_t, as follows:

$$p_t = p_{t-1} + \alpha p_{t-1} \frac{(n_{b,t-1} - n_{s,t-1})}{N}, \tag{2}$$

where $n_{b,t}$ is the number of risk assets bought at t, $n_{s,t}$ is the number of risk assets sold at t, N is the total number of stocks, and α is a coefficient of price fluctuation.

In this situation, when the coefficient of price fluctuation, α, is zero, eq. 3 is satisfied.

$$p_t = p_{t-1} \tag{3}$$

This equation means that the price of marketable assets is fixed at any time. In this study, a single kind of marketable asset is used.

Algorithm to buy and sell bank assets The algorithm to buy and sell marketable assets is based on the local trader of Torii[9]. The logical price of marketable assets at a time point t, p_t^{s*}, is exogenously given by Eq. 4.

$$p_t^{s*} = p_{t-1}^{s*} + \mu_{s*} p_{t-1}^{s*} + \sigma_{s*} p_{t-1}^{s*} \Delta W_{t-1}^{s*} \tag{4}$$

In eq. 4, μ_{s*} means the average logical price, σ_{s*} means the standard deviation of the logical price, and ΔW_{t-1}^{s*} means a Wiener process.

Local traders determine the expected return by using Eq. 5, in which the weighted average of the next three items is calculated.

$$\hat{r}_t^{i,s} = \frac{1}{w_F^i + w_C^i + w_N^i} \left(w_F^i F_t^{i,s} + w_C^i C_t^{i,s} + w_N^i N_t^{i,s} \right) \tag{5}$$

In Eq. 5, w_F^i, w_C^i and w_N^i are the weights of the "fundamental term", "chart term", and "noise term." Each weight is determined for each bank by using a random number so that each will have its own trading strategy.

The expected price at a time point, $t + \tau^i$, $\hat{p}_{t+\tau^i}^{i,s}$ is determined using eq. 6 and $\hat{r}_t^{i,s}$, which is calculated in eq. 5.

$$\hat{p}_{t+\tau^i}^{i,s} = p_t^s \exp\left(\hat{r}_t^{i,s} \tau^i \right) \tag{6}$$

1. Fundamental term
   In this term, the price is predicted by considering the difference between the logical price and market price.
$$F_t^{i,s} = \frac{1}{\tau^{s*}} \ln\left(\frac{p_t^{s*}}{p_t^s}\right) \tag{7}$$
   In Eq. 7, $F_t^{i,s}$ is the fundamental term of a stock s which a bank i holds, p_t^{s*} is the logical price at a time point t, p_t^s is the market price at a time point t, and τ^{s*} is the average regression speed, which is the speed at which the market price approaches the logical price when the market price is largely different from the logical price.
2. Chart term
   This term predicts prices by considering the tendency of the market price before the current step.
$$C_t^{i,s} = \frac{1}{\tau^i} \sum_{j=1}^{\tau^i} \ln \frac{p_{t-j}^s}{p_{t-j-1}^s} \tag{8}$$
   In eq. 8, $C_t^{i,s}$ is a chart term of a stock s which a bank i holds, $/tau^i$ is the length for which the tendency of the market price is dealt, and p_t^s is the market price at time point t.
3. Noise term
   The noise term of a stock s a bank i holds, $N_t^{i,s}$, follows a normal distribution of $N(0, \sigma_\epsilon)$.

The expected price, which is calculated in eq. 6, $\hat{p}_{t+\tau^i}^{i,s}$ is compared to the market price at t, p_t^s. When $\hat{p}_{t+\tau^i}^{i,s}$ is larger than p_t^s, the bank buys a stock, and when $\hat{p}_{t+\tau^i}^{i,s}$ is smaller than p_t^s, the bank sells a stock. Each bank performs a transaction, following the above algorithm.

2.4 Bank Bankruptcy

Judging whether banks are bankrupt Whether banks go bankrupt or is determined by eq. 9.

$$CAR_t = \frac{Networth_t}{VaR * StockNum_t} \tag{9}$$

In this equation, CAR_t is the capital adequacy ratio at t, $Networth_t$ means net worth at t, and $StockNum_t$ is the number of stocks the bank holds.

VaR stands for value at risk, which is a quantitative index to measure the risk of a certain stock. The denominator of Eq. 9 is the product of the risk of the stock and the number of stocks; i.e., it is the total risk. When VaR falls below 0.04, the bank goes bankrupt.

Dealing with bankrupt banks When a certain bank goes bankrupt, other banks which have lent to it can no longer collect debts from it. On the other hand, the bankrupt bank does not have to clear its debts. For the sake of brevity, the marketable assets of the bankrupt bank are not sold to the market. Namely, each bank continues to hold marketable assets after it goes bankrupt, but it can no longer buy and sell them.

3 Results and Discussion

As mentioned in section 1.2, we studied chain bankruptcy by using an agent simulation in which two parameters were varied:

1. the volatility of marketable assets that banks hold and
2. the type of inter-bank lending and borrowing network.

The experiments were conducted 100 times for each parameter, and the average number of bankrupt banks was calculated.

3.1 Effect on fluctuation of marketable assets

The coefficient of price fluctuation α, defined in eq. 2, was set as the parameter. Table 1 shows the six patterns of α tried in the simulations.

Table 1. Values of coefficient of price fluctuation α

α	0	0.25	0.5	0.75	1.0	2.0

We used core-periphery networks in which the large banks had 30 links and the smaller banks had ten links.

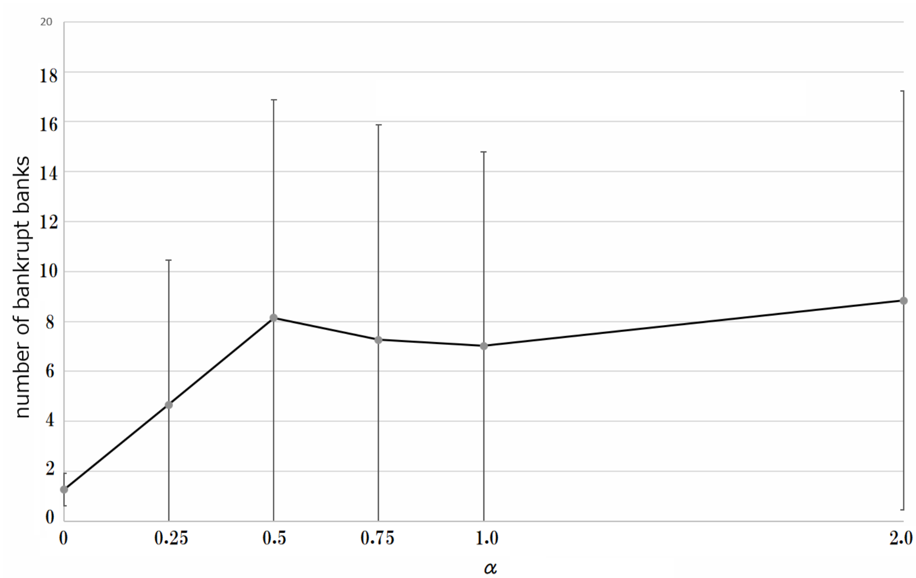

Fig. 2. Relationship between coefficient of price fluctuation and the number of bankrupt banks
The length of error bars corresponds to each sample variance.

The results in Fig. 2 indicate that as the coefficient of price fluctuation grows, the number of bankrupt banks increases.

Comparing the cases in which α is 0 and α is 1.0 or 2.0, we can see that as soon as the price fluctuates, the number of bankrupt banks increases. This is because the fluctuation of marketable assets enlarges the risk that banks hold. In particular, the number of bankrupt banks rapidly increases between the case in which α is 0 and the case in which α is 0.5. However, the average number of bankrupt banks does not change much between the cases in which α is 0.5 and α is 2.0. This is because when α exceeds a certain limit, almost all banks holding marketable assets over a certain amount go bankrupt.

3.2 Effect on inter-bank lending and borrowing networks

The results for each network are described below.

Core-periphery networks In the core-periphery networks, the number of links determined in step 5 in section 2.2 was set as the parameter.

Table 2 shows the number of links of large banks and smaller banks.

Table 2. Value of parameters in core-periphery networks

Number of links of large banks	20 30 50
Number of links of smaller banks	5 8 10 20

We conducted a round-robin on the number of large banks versus the number of smaller banks, as shown in Table 3.

Table 3. Patterns of parameters in core-periphery networks

	large banks	smaller banks		large banks	smaller banks
pattern 1	20	5	pattern 7	20	10
pattern 2	30	5	pattern 8	30	10
pattern 3	50	5	pattern 9	50	10
pattern 4	20	8	pattern 10	20	20
pattern 5	30	8	pattern 11	30	20
pattern 6	50	8	pattern 12	50	20

Figure 3 shows the result.

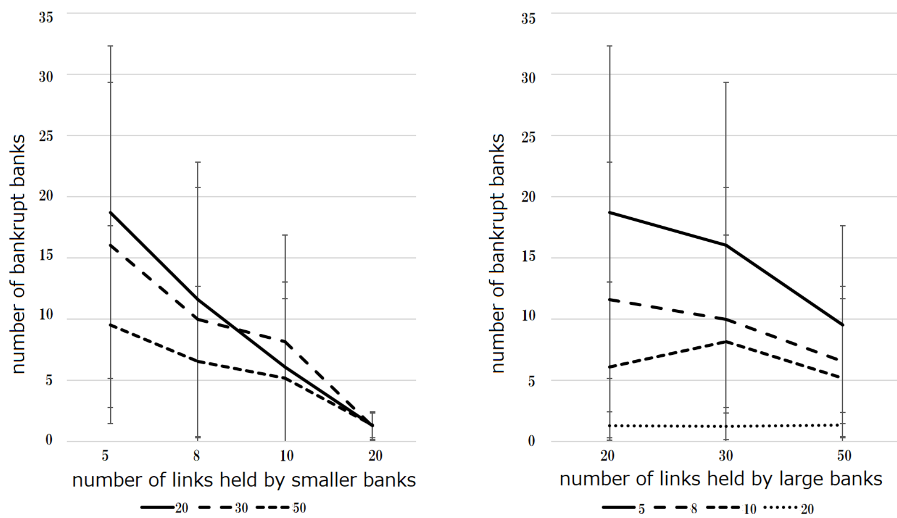

Fig. 3. Relationship between the number of links and the number of bankrupt banks in core-periphery networks
The length of error bars corresponds to each sample variance.

As the number of links increases, the average number of bankrupt banks decreases, except for certain patterns. The exceptions are patterns 7 to 9 and 10 to 12, where the numbers of links are 10 or 20. The reason for this is explained in the next section.

Scale-free networks In scale-free networks, the number of links of the added banks in step 2 in section 2.2 was set as the parameter. Table 4 shows the number of links of added banks.

Table 4. Patterns of parameters in scale-free networks

number of links of added banks	2 3 5 8 10

Figure 4 shows the result.

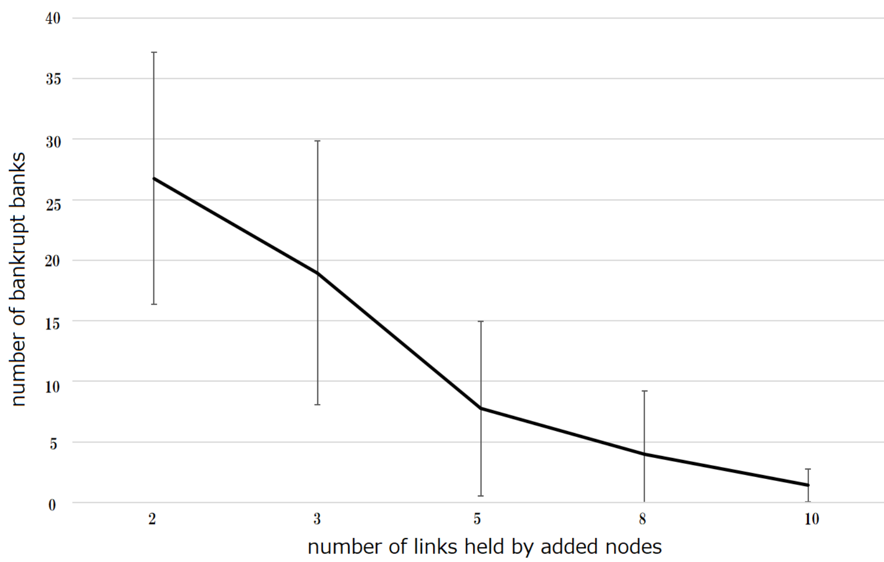

Fig. 4. Relationship between the number of links and the number of bankrupt banks in scale-free networks
The length of error bars corresponds to each sample variance.

The number of bankrupt banks decreased in a monotone manner as the number of links of added banks and the number of links in the whole network increase.

Random networks In random networks, the number of links that large banks and smaller banks have is set as the parameter. Our purpose for studying random networks was to know the basic behavior of the number of bankruptcy banks as the number of links increased, so we examined four different parameters shown in Table 5.

Table 5. Parameter patterns in random networks

	large banks	smaller banks		large banks	smaller banks
pattern 1	6	3	pattern 3	20	10
pattern 2	10	5	pattern 4	50	20

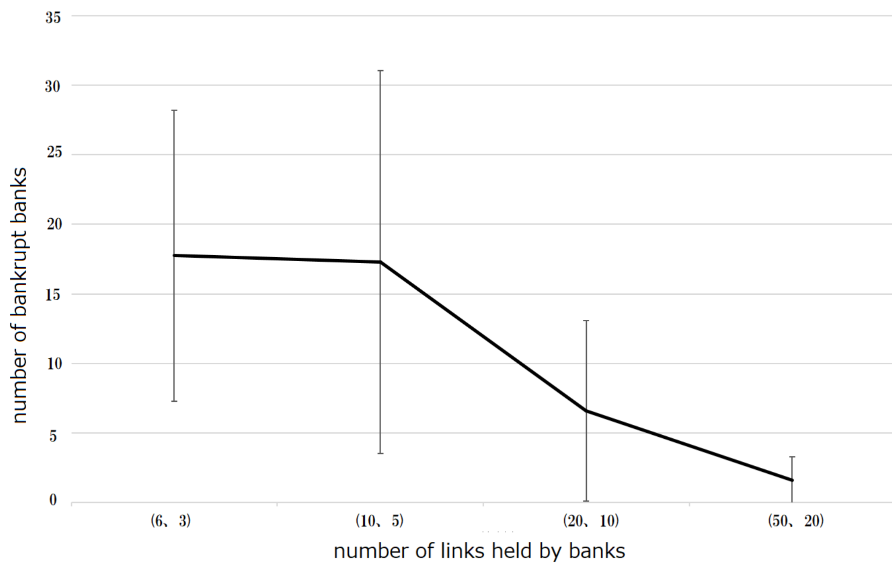

Fig. 5. Relationship between the number of links and the number of bankrupt banks in random networks
The length of error bars corresponds to each sample variance.

The results in Fig. 5 show that as the number of links increases, the number of bankrupt banks decreases.

To see the reason for this result, we note that as the number of links increases, the number of bankrupt banks decreases in the random networks and scale-free networks. In these simulations, the sum of all the assets that each bank holds is set to a certain range, so no matter how large or small the number of links of each bank is, the loan amount is almost the same. For example, let us consider cases in which the number of links of a certain bank X is 3 (pattern I) or 20 (pattern II).

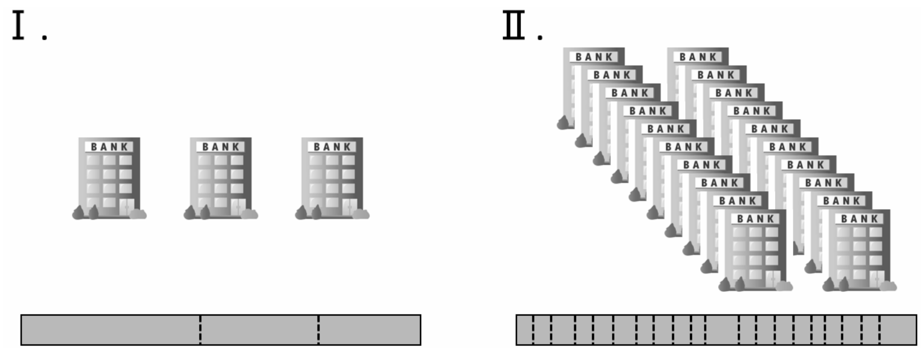

Fig. 6. Schematic view of loans

Figure 6 shows a schematic view of the loan amounts. The length of the bar at the bottom represents the loan amount; it is almost constant in patterns I and II. The bar is partitioned; each square corresponds to the amount of money which bank X lent to each bank.

Here, let us suppose a bank to which bank X lent money goes bankrupt (Fig. 7).

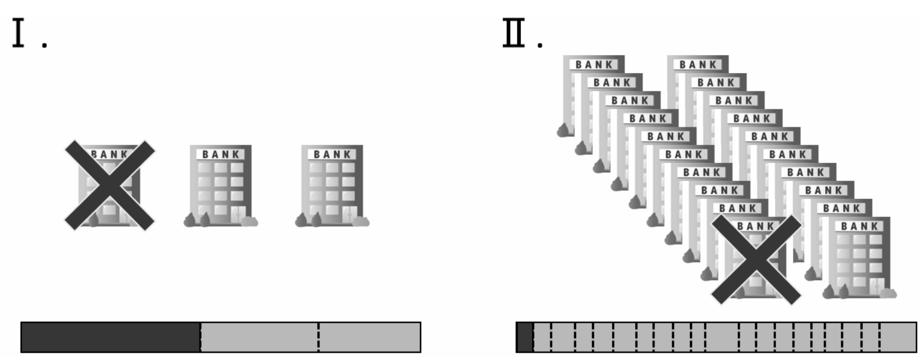

Fig. 7. Schematic view of loans when a certain bank goes bankrupt

In this situation, the debts which bank X can not collect varies depending on patterns I and II. The debts that bank X cannot collect in pattern I is larger than in pattern II. Therefore, when the number of links is small and a client goes bankrupt, the damage is larger. This is the reason for the result that the number of bankrupt banks decreases as the number of links increases.

Moreover, as the number of links increases, the number of bankrupt banks decreases, but in some cases of the core-periphery networks, the number of bankrupt banks did not decrease in a monotone manner. This happens when the number of links of smaller banks is 10 and 20. Let us consider the behavior in that case. When the number of links of the smaller banks is large, the change in the number of links of the large banks does not affect the number of bankrupt banks. On the other hand, when the number of links of the large banks is large, as the number of links of the smaller banks increases, the number of bankrupt banks decreases.

Here, we can distinguish three types of link.

1. links which connect large banks to large banks
2. links which connect large banks to smaller banks
3. links which connect smaller banks to smaller banks

In this network model, because every large bank is connected each other, type 1 does not increase even if the number of links changes. So, when the number of links of the large banks increases, only type 2 increases. Also, when the number of links of the smaller banks increases, type 2 and 3 increase.

With all of the above things considered, when the number of links is large in core-periphery networks, the change in the number of type-2 links does not affect the number of bankrupt banks, whereas an increase in the number of type-3 links makes the inter-bank networks stable.

4 Conclusion

4.1 Conclusion

We studied the effect on chain bankruptcy of varying

1. the volatility of risk assets that banks hold, and
2. the type of inter-bank lending and borrowing network.

The conclusions are as follows:

1. As the coefficient of price fluctuation α increases, the number of bankrupt banks increases.
2. As the number of links increases, the number of bankrupt banks decreases.

Regarding conclusion 1, while the number of bankruptcies rapidly increases between the case in which α is 0 and the case in which α is 0.5, it is almost constant when α is larger than 0.5.

Regarding conclusion 2, it can be said that when banks lend the almost same amount of money, they should divide up that amount and lend those portions to more banks so as to disperse the risk.

4.2 Future plans

We should consider the effect of connecting the inter-bank network to a supply chain, i.e., a network of companies.

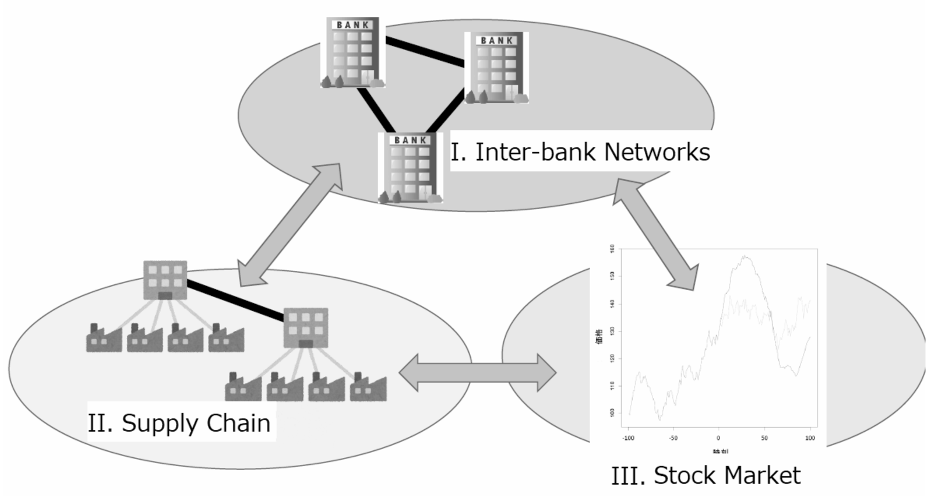

Fig. 8. Framework of future plans

By adding a supply chain to an inter-bank network, we can consider banks loaning to companies and price fluctuations due to company activities. We want to make a framework in which "banks", "companies", and "markets" are integrated and thereby observe the effect of a bankruptcy of a certain company on the whole economic system.

As a first step, we will examine the supply chain and combine it with the inter-bank network. Regarding supply chains, there are studies of Miura[5]. In Miura's research[5], a growth model of the enterprise' s trading network is set up, and features of the supply chain are explained. Regarding risk asset markets, we also plan to build an artificial market based on Torii's research[10].

4.3 Acknowledgements

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